A review of belief changes in two approaches

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Abstract

Belief is one of the fundamental concept that people use everyday. The study on belief change has been an active topic in both computer science and philosophy. In this article, we will start from the concept of belief and then analyze how the topic has been discussed in computer science and dynamic epistemic logic. We will make a discussion on comparing the two approaches.

Key words: belief , belief revision, belief update, belief upgrade, AGM model

1 Introduction

Belief is an important daily notion that people employ to express certain kind of attitude toward the truth of something. We mostly believe that the sun will raise also tomorrow. Ancient Chinese believed their Emperor was designated by Heaven. Italians believe that their food is the best of the world. In philosophy, the concept of belief plays an important role in both philosophy of mind and epistemology. Forming beliefs is one of the most important features of the mind. Generally, belief is used by contemporary analytic philosophers as "propositional attitude", i.e. the mental state corresponding to the attitude one has toward a proposition. For instance, in a sentence like "Anna believes that her husband is working at the office.", "her husband is working at the office" is the proposition; "believes" is the attitude; and "Anna" is the sentient being in the sense of the subject of the attitude. However, here we would like to restrict our discussion on belief change, focusing in particular on the dynamic view toward this concept, while leaving aside a more thorough exploration on modern philosophical discussions on the concept of belief.

People's beliefs do not always stay static, but change with respect to different reasons. According to the definition given above, we will soon realize that this means that, due to some reason, the subject changes the propositional attitude toward the truth of something. The study on belief change is relatively new. Indeed, it has been explored and soon thrived as a research field only since 1980s. It was aimed at serving as the connection of two research fields, that is, computer science and philosophy. In computer science field, inspired by artificial intelligence and the need of studying the database update, researchers started to develop sophiscated models. The milestone for this topic is AGM model. On the other hand, for philosophy, the study of belief change has been a subject of philosophical reflection since antiquity. For logic, studying belief and belief change is one of the important topic of Dynamic Epistemic Logic (DEL), in which belief has been axiomatized and its dynamification has been discussed a lot.

From the passages above, we can soon notice that the topic of belief change has been discussed in the fields of computer science and logic. Even though researches in these two fields share the same topic, their methodologies are quite different. We believe, though, that the study of the connections and differences between these two fields inasmuch as belief change is concerned could deserve an analysis. Following, in section 2, we will introduce AGM model, which is central to belief revision theory in computer science. Afterwards, in section 3, we will analyse the fundamental idea behind belief update and upgrade in DEL field. In section 4 we make the comparison. Finally, in section 5 we will draw some conclusions.

2 AGM Model

AGM model has a profound influence in the study of belief base updating in computer science. It is a central model to the belief revision theory. The word "AGM" is named after three originators, Carlos Alchourron, Peter Gardenfors, and avid Makinson. Their joint paper,i.e. "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions", published in Journal of Symbolic Logic 1985, has soon been a starting point for the subsequent studies on belief changes.

In computer science, in order to attain a better understanding of belief changes in database, we need take three chief concerns into account: (1) how are the belief in database represented (2)what is the relationship between the elements explicitly represented in the database and the belief that may be derived from these elements (3) how are the choices concerning what to retract made. In AGM model, an agent's beliefs are represented by a set that is closed under logical consequence. To construct the whole model, we need to select a formal language, and then represent beliefs as sentences in the language. In fact, as we study both belief sentences and belief sets, we need make a distinction in the language. We represent belief sentences by lowercase letters p, q, r...We represent belief sets by capital letters A, B, C...The lowercase letters mentioned above is connected by Boolean operators: negation($\neg$) ,conjunction($\wedge$), disjunction($\vee$), implication($\rightarrow$), and equivalence($\leftrightarrow$).$\top$denote an arbitrary tautology,$\bot$denote an arbitrary contradiction. It is naturally for us to regard belief sets as sets of beliefs, but it is not enough. If we simply put a bundle of beliefs into a belief set, we would probably make a mess. Some beliefs are contrary to each other, like p and $\neg p$. Some beliefs are logically following from the other beliefs. The belief sets representing mind states are these sets that are logically closed under logical consequences.

Definition(Belief Set)

A set K of sentences in L is a belief set if and only if it is logically closed under logical consequences, that is, If K is logically entails ψ,then $\psi \in K$.

It is obvious that if a belief set K is inconsistent, by classical logic, it would contain all the sentences in the language L, then there is only one belief set K, denoted as $K_{\perp}$. At the same time, by definition we can see that belief sets contain all the sentences that are all accepted in this set. Moreover, belief sets satisfy following properties:

Let K denotes an arbitrary $Cn(K)$, it is the set of logical consequences of K. Moreover the function Cn satisfies conditions below:

(1) Inclusion $K \subseteq Cn(K)$

(2) Monotony: If $K \subseteq Q$,then $Cn(K) \subseteq Cn(Q)$

(3) Iteration: $Cn(K) = Cn(Cn(K))$

K is a belief set if and only if $K = Cn(K)$

2.1 Belief change

Generally, when a new belief p is added into an arbitrary set A of beliefs, belief change occurs. In a broader sense, we can distinguish belief changes into three kinds: expansion, contraction, revision.

First, the idea behind expansion of a set A of beliefs is simple. We expand the a set A of beliefs by directly putting a new belief p into A. We do not need to care about whether p is consistent with A or not. The operation of expansion is denoted by $+$, and the resulting set of belief is denoted as$A + p$.

Secondly, the general idea of performing contraction on a set of beliefs A is to delete some elements in A. The outcome of contracting A by p should be a subset of A that does not imply p. Contraction on a belief set K with respect to p can be denoted as $K \div p$. The outcome of performing contraction should be a subset of K that does not imply p anymore.

Finally, if a new belief p, which is inconsistent with a belief set, is added into the belief set, then belief revision occur. If we just change belief sets just by adding new elements into them, regardless of logical consistency, then the resulting sets would be probably not a consistent belief set at all. The problem is concentrated on how we can move from a consistent belief set to a new consistent belief set. Revision on a belief set K means to add the new belief p into the belief set K without losing the consistency as well as logical closed property. To make it work, we need to contract several beliefs first in the original belief set K which contradict p, and then expand the contracted set with respect to p. The operator for belief revision is denoted as $K * p$. $K * p = (K \div \neg p) + p$

2.2 Basic AGM postulates.

AGM postulates are central to AGM model. These postulates determine how exactly the revision operation works. These basic AGM postulates are:

Closure $K * p = Cn(K * p)$ This postulate means that after revision, the resulting belief set is logically closed.

Success: $p \in K * p$ This postulate means that after revision, belief p is definitely added into the belief set$K$.

Inclusion: $K * p \subseteq K + p$ This postulate means that after revision the resulting belief set $K * p$ is a subset of the belief set $K + p$, which is the resulting set after expansion.

Vacuity: If$\neg p \notin K$,then $K * p = K + p$ This postulate means if belief set K does not contain belief $\neg p$, which is contrary to belief p, then the resulting belief sets for expansion of K with respect to p and revision of K with respect to p are just the same one.

Consistency:$K * p$ is consistent if p is consistent. This postulate means that if p is already consistent, then resulting belief set $K * p$ is for sure consistent. $K * p$ is inconsistent only if p is inconsistent or K is inconsistent.

Extensionality: If $(p \leftrightarrow q) \in Cn(\emptyset)$,then $K * p = K * q$. This postulate means if p and q are logically equivalent, then revision of belief set K with respect to p and revision of belief set K with respect to q will have the same result.

Belief revision meets these six postulates are called partial meet revision. It is the standard operation of revision in the AGM model.

In addition, two supplementary postulates are part of the standard repertoire:

Superexpansion: $K * (p \wedge q) \subseteq (K * p) + q$

Subexpansion: If $\neg q \notin Cn(K * p)$,then $(K * p) + q \subseteq K * (p \wedge q)$

A belief revision operator that satisfies all eight postulates is the full meet revision, which has been considered to be too conservative.

3 Belief changes in DEL

Studying belief changes is one of the important topics in dynamic epistemic logic (DEL) of information change. Traditionally the concept of belief is closely connected to the concept of knowledge in epistemology. For the topic of belief change, knowledge influence belief as well. Generally we distinguish two kinds of triggers responsible for information change: hard information and soft information. On one hand, if information can change what an agent knows, then it is regarded as hard information, such as public announcement $!p$ of a fact p. Around hard information, we can derive logic of information update and knowledge change. On the other hand, if information can not change what an agent knows, but affect the agent's beliefs, preferences and so on, then it is regarded as soft information. Around soft information, we can derive logic of information upgrade.

Generally, in DEL scenario, if we would like to discuss a certain concept and its dynamification, the first step is to set a static base logic and its matching models. At the same time we will try to make a complete axiom system for this concept. Then the second step is to analyze some relevant informational events to check if they can serve as triggers for changing these models. Finally the third step is to make dynamic expansion on the static base with respect to certain triggers. In this way, we will attain a dynamic logic. At the same time, we will try to design a complete axiom system for dynamic logic. The usual method is to add dynamic reduction axioms into the original axiom system. Here, reduction axioms can describe how exactly the dynamic operators work, and they serve as reducting any dynamic formula into its static counterpart. With the help of reduction axioms, we can prove the completeness of the dynamic logic system.

Next, we will roughly demonstrate how belief changes are characterized following the above methodology:

In DEL, belief is defined as a modal operator B, and $B_i\varphi$stands for that an agent i believe that φ. The corresponding language is similar to basic modal logic. We just change the necessity operator $\Box$into belief operator$B$. Next, we will define a model and give the semantics. A model for the logic of belief is a tuple defined as $M = (W, \{\leqslant_i\}_{i\in I}, V)$:$W$ is a set of possible worlds. $\leqslant_i$is a binary relation, which is read as "at least as plausible as".V is truth assignment. Next, we give the semantics: $M, s \models B_i\varphi$if and only if $M, t \models \varphi$,for all worlds t which are minimal for the ordering $\leqslant_i$. It means that if $\varphi$is true on the most plausible worlds, then agent i believes that φ. Usually it is not enough if we only define belief operators, because we need to define conditional belief operator, which will function in deduction axioms later. The operators for conditional belief is denoted as B^ψ,ψ is the condition.$B_i^\psi\varphi$ is read as "agent i believes φon condition of ψ". The semantics for conditional belief is as follows: $M, s \models B_i^\psi\varphi$if and only if $M, t \models \varphi$for all world t which are minimal for the ordering $\leqslant_i$in the ψ-worlds. It means that if $\varphi$is true on the most plausible worlds on which $\psi$is true, then agent i believes φon condition of ψ.

Now we move to the step of belief change. We distinguish two kinds of belief changes: belief update and belief upgrade.

Belief update occurs if hard information is adopted, such as public announcement. The basic mechanism of belief update is the same as that of PAL. We denote $[!p]$ as update operator.When hard information p is adopted, all the $\neg p$-worlds will be eliminated from W. The logic of belief under update is axiomatized completely by (1) any complete static logic for the model class chosen, (2) the PAL reduction axioms for atomic facts and Boolean operations, (3) the reduction axiom for beliefs: $[!p]\, B_i\varphi \leftrightarrow (p \to B_i^p([!p]\,\varphi))$.

For belief update, it is noticeable that this method is too strong. As long as information p has been adopted, all the $\neg p$-worlds disappears, leaving only the p-worlds. Thus lots of information contained in the $\neg p$-worlds also lost with respect to the disappearance of all$\neg p$-worlds. The unpleasant thing is we cannot go back any more under this kind of update. An alternative choice is belief upgrade. Belief upgrade occurs if soft information p is adopted. We denote $[\Uparrow p]$for upgrade operator. For belief upgrade, we do not need to eliminate all the $\neg p$-worlds. Instead, we can keep all the $\neg p$-worlds and change the ordering between possible worlds. Before upgrade, there might be some $\neg p$-worlds are better than p-worlds. When soft information p is adopted, all the p-worlds now become better than all the $\neg p$-worlds. Apart from this situation, all the other relations keep just the same as before. Belief upgrade can also be completely axiomatized. Actually there are two complete dynamic logics for belief upgrade.

4 Comparison

In previous sections we have analyzed belief revision in AGM model and belief update and upgrade in DEL. Even though two approaches deal with belief changes , they do it in quite different way:

(1) Their theoretical background are different. Belief revision is founded in computer science field. It is designed to solve concrete practical problems in updating database. While for belief update and upgrade in DEL, it is designed to delicately capture the concept of belief.

(2) Belief revision attempts to decide what beliefs should be discarded when an agent adopts a new belief. The criterion for us to follow is that an agent should make minimal changes in his belief set in order to accept a new belief. Belief revision focuses on what changes will happen to a belief set.Belief update and upgrade attempts to decide when a trigger occurs, how it changes the model and how the static model and the updated/upgraded models are related. In this sense belief update/upgrade focuses on what changes will happen to models and axiom systems.

(3) Belief revision deals with belief change in an abstract way. It does not provide with concrete mechanisms for us to select which belief should be discarded and which belief should be kept. Rather, it merely provide with postulates like AGM postulates to make certain what is an appropriate revision. Belief update/upgrade does the opposite. Belief update/upgrade determines concrete methods to follow.Triggers as hard information and soft information are responsible for belief changes. For belief update, when hard information $[!p]$is adopted, we will eliminate the worlds that contain contrary information p. For belief upgrade, when soft information $[\Uparrow p]$is adopted, we will change ordering between worlds.

(4) In belief revision, when a new belief p occurs, we will accept this belief and accommodate it into the original belief set. In belief update/upgrade, we do not accept triggers. Triggers are not certain beliefs. Triggers serve as information that change the possible worlds model.

(5)The ultimate solution to the problem about belief revision is to develop algorithms for computing appropriate revision and contraction functions for an arbitrary belief system.To realize this idea, we try to capture the intuition that an agent should make minimal changes in his beliefs in order to accommodate a new belief. But for belief update/upgrade, our focus is to model changes and try to make a balance between a static model and a updated/upgraded model. Yet it does not mean that belief changes in DEL pay no attention to information loss. We have noticed that after updating, agents' knowledge may probably change with respect to world-elimination operation. Belief upgrade due to soft information can be regarded as improvement to avoid that problem. It only changes the plausibility relation between worlds and will not affect agents' knowledge.

5 Conclusions

The study on belief change has been an active topic in both computer science and philosophy. In this article, we have mainly analyzed how the topic has been discussed in computer science and dynamic epistemic logic. We have started from the concept of belief, and then mainly reviewed two approaches to describe belief change: belief revision in computer science, and belief update and upgrade in DEL. Finally we have made a discussion on comparing the two approaches. By comparison we have found the two approaches vary largely from each other at five points.

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